

GENERIC ATTACKS ON DUPLEX-BASED AEAD MODES

SMALL CYCLES AND LARGE COMPONENTS

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We call $\mu(x)$ and $\lambda(x)$ the cycle length and tail length respectively

RELEVANT VALUES

DEFINITION (ν -COMPONENT)

let $0 < \nu < \frac{1}{2}$. A ν -component is a component that has a cycle of size at most $n^{\frac{1}{2}-\nu}$.

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DEFINITION (v -COMPONENT)

let $0 < v < \frac{1}{2}$. A v -component is a component that has a cycle of size **at most** $n^{\frac{1}{2}-v}$.

DEFINITION ((s, v) -COMPONENT)

let $0 < v < \frac{1}{2}$ and $0 < s < 1$. A (s, v) -component is a component whose size is **greater or equal to** ns and whose cycle is of size **at most** $n^{\frac{1}{2}-v}$.

PREVIOUS WORKS

- ▶ It is known that $\mu(x)$ and $\lambda(x)$ are both on average $\sqrt{\pi n/8}$. See famous "Random mapping statistics, Flajolet and Odlyzko" in 1989

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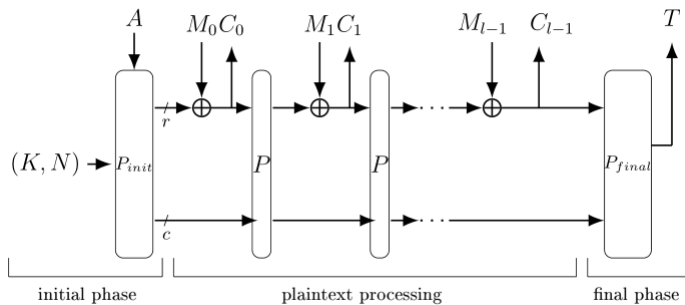
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De Laurentis, Crypto 1987, "Components and Cycles of a random function"

DUPLEX AEAD



SECURITY OF DUPLEX

Simplified Beyond conventional security in sponge-based authenticated encryption modes [Jovanovic, Luykx, Mennink, Sasaki, Yasuda, JoC 2019]

$$\mathcal{T} \ll \min\left\{2^{\frac{b}{2}}, \frac{2^c}{\alpha}, 2^k\right\} \text{ and } q_d \ll 2^\tau$$

where, $\alpha < r$, where q_d is the number of forgery attempts.

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So duplex construction is proven for $2^{\frac{c}{2}}$, and known generic attacks are in $\frac{2^c}{\alpha}$

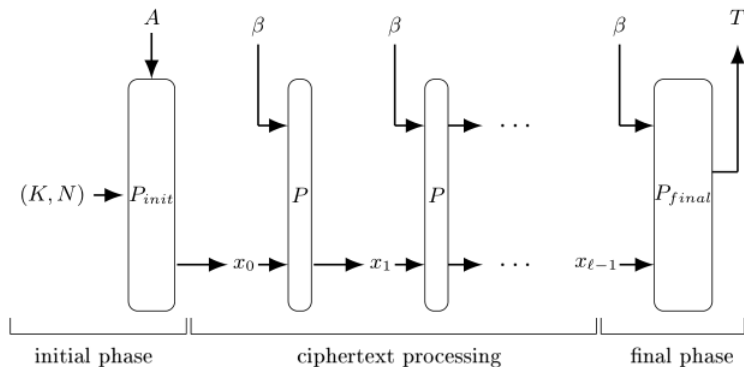
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Let $C_\beta^\ell = \beta_\ell = \underbrace{\beta || \dots || \beta}_\ell$. Then the decryption of C_β^ℓ corresponds to the iteration of

$$f_\beta : \mathbb{F}_2^c \longrightarrow \mathbb{F}_2^c \\ x \longmapsto [P(\beta || x)]_c.$$



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Online : input (N, A, C, T) with N, A possibly different and $C = C_\beta^\ell$ with $\ell = \gamma 2^{\frac{c}{2}}$. And T being derived from a value of the cycle of f_β ($n^{\frac{1}{2}-v}$ possibilities at most)

PRECOMPUTATION

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- ▶ (s, v) costs too much, so we use an approximation (CLT)

- ▶ Input N, A, C_{β}^{ℓ} and a proportion of possible tags (with respect to cycle's values)

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Complexity $O\left(2^{\frac{3c}{4}}\right)$

EXPERIMENTAL VERIFICATION

Statistics verified up to small c values.

SPECIFIC MODES AND PADDING

Key recovery is possible and attack applicable to several proposals :

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- ▶ Motorist : Keyak

WHAT FRUSTRATES THE ATTACK

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- ▶ Use a ρ -like application (Beetle, Subterranean)